Muon g-2 and Physics Beyond Standard Model

Atanu Nath

Istituto Nazionale di Fisica Nucleare - Sezione di Napoli, Naples, Italy.

On behalf of the Fermilab muon g-2 collaboration.

Department of Physics, Assam University Silchar

Silchar, India

August 21, 2018

One slide on me

- I am from Lala, Hailakandi.
- Did my B.Sc (Physics) from the Dept. of Physics, ***G. C. College***, Silchar (2007).
- M.Sc from the ***S. N. Bose National Centre for Basic Sciences***, Kolkata (2010).
- Two years of graduate school course work at ***Tata Institute of Fundamental Research***, Mumbai after M.Sc.
- Ph.D, ***University of Naples "Federico II"***, Naples, Italy, on modelling non-perturbative QCD techniques of the rare Kaon decays.
- Post Doctoral Fellow (*Assegni di Ricerca*), ***INFN (National Institute of Nuclear Physics)***, Naples, Italy in collaboration with Fermilab, USA.

What is g and what's with that minus 2?

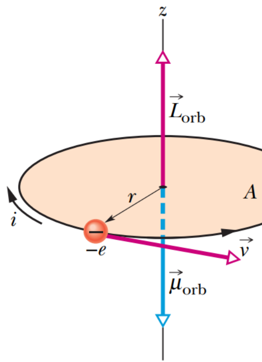
Consider a very crude model of an electron orbiting in a circle of radius r with a speed v. The magnetic moment $\boldsymbol{\mu}_{orb}$ will be given by the resulting current times the area of the circle:

$$\begin{aligned} \boldsymbol{\mu}_{orb} &= i\,\boldsymbol{A} \\ &= \frac{-e}{2\pi r/v}\,\pi r^2\,\hat{A} = -\frac{e}{2m}\boldsymbol{L}_{orb} \end{aligned}$$

where $\boldsymbol{L}_{orb} = \boldsymbol{r} \times m\boldsymbol{v}$.

This expression also holds for a fundamental particle with intrinsic angular momentum $\boldsymbol{S}$ and charge Q provided we correct it with a factor g:

$$\boldsymbol{\mu} = g\,\frac{Q}{2m}\,\boldsymbol{S}$$



Such a magnet when placed in a magnetic field experiences a torque that gives rise to a potential energy

$$U = -\boldsymbol{\mu}\cdot\boldsymbol{B}$$

The 2 in $g - 2$

In relativistic QM, the wavefunction of a spin $1/2$ fundamental particle ℓ with charge and mass e_ℓ and m_ℓ respectively, interacting with an external EM-field $\mathcal{A}_\mu(x)$ obeys the Dirac's equation (in minimal coupling).

$$i\,\frac{\partial \psi}{\partial t} = \left[\boldsymbol{\alpha}\cdot\left(-i\,\boldsymbol{\nabla} - e_\ell \boldsymbol{\mathcal{A}}\right) + \beta m_\ell + e_\ell \mathcal{A}_0\right]\psi$$

in the non-relatvistic limit this becomes the Pauli's equation with 2-component spinor ϕ

$$i\,\frac{\partial \phi}{\partial t} = \left[\frac{\left(-i\,\boldsymbol{\nabla} - e_\ell \boldsymbol{\mathcal{A}}\right)^2}{2m_\ell} - 2\,\frac{e_\ell}{2m_\ell}\boldsymbol{S_\ell}\cdot\boldsymbol{B} + e_\ell \mathcal{A}_0\right]\phi$$

where $\boldsymbol{S}_\ell = \boldsymbol{\sigma}/2$, is the spin of the particle.
Comparing the second term with $U = -\boldsymbol{\mu}\cdot\boldsymbol{B}$ we see that Dirac's equation predicts $g = 2 \implies g - 2 = 0$ for leptons.
But is it??

History of $g - 2 > 0$

Lamb Shift (1947): According to Dirac equation the energy difference between $2S_{1/2}$ and $2P_{1/2}$ levels of the Hydrogen atom should be zero.

Celebrated paper...

PHYSICAL REVIEW VOLUME 72, NUMBER 3 AUGUST 1, 1947

Fine Structure of the Hydrogen Atom by a Microwave Method* **

WILLIS E. LAMB, JR. AND ROBERT C. RETHERFORD
Columbia Radiation Laboratory, Department of Physics, Columbia University, New York, New York
(Received June 18, 1947)

THE spectrum of the simplest atom, hydrogen, has a fine structure[1] which according to the Dirac wave equation for an electron moving in a Coulomb field is due to the combined effects of relativistic variation of mass with velocity and spin-orbit coupling. It has been considered one of the great triumphs of Dirac's theory that it gave the "right" fine structure of the energy levels. However, the experimental attempts to obtain a really detailed confirmation population and the high background absorption due to electrons. Instead, we have found a method depending on a novel property of the $2^2S_{\frac{1}{2}}$ level. According to the Dirac theory, this state exactly coincides in energy with the $2^2P_{\frac{1}{2}}$ state which is the lower of the two P states. The S state in the absence of external electric fields is metastable. The radiative transition to the ground state $1^2S_{\frac{1}{2}}$ is forbidden by the selection rule $\Delta L = \pm 1$. Calculations of Breit and Teller[4]

But the *Nobel* winning experiment based on atomic beam-microwave technique developed by Willis Lamb and carried out by Lamb and Retherford found a relative shift of 1058 MHz suggesting g slightly greater than 2! This result fueled the development of modern QED, the most successful theory in science so far.

RQM is not complete!

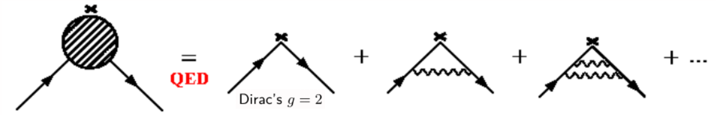
- Relativistic QM is not complete.
- Soon Schwinger, Tomonaga, Feynman and others started developing **Quantum Electrodynamics (QED)**, that soon earned the reputation of the "**most accurate theory of nature**" so far and it still stands so.
- Feynman developed his diagramatic method that simplified super-complicated calculations of QED.
- Schwinger despised this approach,

  "*Like the silicon chips of more recent years, the Feynman diagram was bringing computation to the masses*".

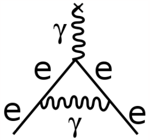
  Thanks to Feynman for that though.

Let's look at the digrams that represent contributions from virtual particles of the **vacuum** contributing to the magnetic moment of a particle.

Anomalous magnetic moment of the electron



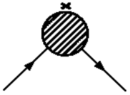
Schwinger's One-loop calculation (1948): Dirac's $g = 2$ corresponds to the lowest order (tree diagram, that is no virtual particles involved!) result in QED, the first order correction (1-loop) was calculated by Julian Schwinger



$$\frac{g_e - 2}{2} = a_e = \frac{\alpha}{2\pi} + \mathcal{O}(\alpha^2)$$

The result wass so profound that it got engraved in his grave.

What's running in the loops?



That blob includes everything that is allowed in nature, leptons, quarks, weak-bosons or something unknown to current physics, any virtual field can interact with the lepton ℓ (that is running in a loop) in question and contribute to its g_ℓ factor. This is precisely why g_ℓ acts as an excellent probe to what's lurking in the vacuum, SM and/or BSM fields.

The anomaly is defined through the quantity $a_\ell = (g_\ell - 2)/2$. Total anomaly can be written as:

$$a_\ell = \underbrace{a_\ell^{QED} + a_\ell^{hadronic} + a_\ell^{weak}}_{\text{Standard Model}} + \underbrace{a_\ell^{BSM}}_{\text{New Physics}} \tag{1}$$

We need to know the SM contribution both theoretically and experimentally with equal precision in order to say something conclusive about the Beyond Standard Model part.

What's running in the loops?

Suppose a virtual particle of mass M_{loop} is running in the loop, contributions to the magnetic moment from such a loop enter as functions of m_ℓ^2/M_{loop}^2

$$a_\ell \sim f\left[\frac{m_\ell^2}{M_{loop}^2}\right]$$

therefore we can guess the mass of an unknown particle from such a loop if we separate all other known contributions from the complete result.

But why μ? What's wrong with e?

As we have seen, loop contributions enter as functions of m_ℓ^2/M_{loop}^2

$$a_\ell \sim f\left[\frac{m_\ell^2}{M_{loop}^2}\right]$$

electron being the lightest lepton, even in a one-muon loop $M_{loop} = m_\mu$ implies (m_e^2/m_μ^2) a contribution $\mathcal{O}\left[10^{-10}\right]$.

Muon is ,

$$\frac{m_\mu^2}{m_e^2} \simeq 43000$$

times more sensitive than electron in sensing a heavy unknown particle. Although a_e has been measured to ~ 0.3 *part per billion*, it is insensitive to any heavy new physics scales.

What's wrong with τ then?

τ is the heaviest lepton therefore m_τ^2/M_{loop}^2 is the biggest for τ, certainly a_τ will be the most sensitive probe to any new physics.

Well it is and the theoretical calculation provides us with a very precise value of

$$a_\tau = 117721(5) \times 10^{-8}$$

but it is so short-lived (10^{-13} seconds) that no practical experiment can be designed with the current technology, current experimental bound is

$$-0.052 < a_\tau < 0.013$$

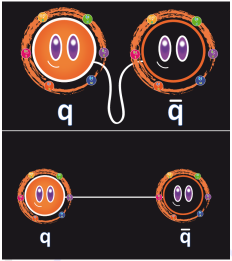
Even the sign is not known experimentally!!!

This leaves us with the only choice μ.

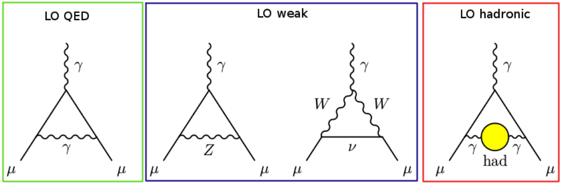
The hadronic issue!

Electron anomalous magnetic moment, as we have seen, has been caluclated so precisely because it's insensitive to heavier particles, therefore just QED calculation is enough. Muon situation is not that fortunate though! This is because,

- QCD has this beautiful (yet nasty!) property called the "***assymptotic freedom***", strength of attraction between two quarks increases as we pull them apart.
- Therefore long-distance (low-energy) calculations involving **quarks** and **gluons** are impossible using direct QCD, they rely on models and experimental data.
- That's why **hadronic** contributions introduce big uncertainties!



The hadronic issue!



From left to right we have **leading order QED**, **weak** and **hadronic** contributions, biggest uncertainty of course enters from the hadronic (quark and gluon loops) contributions, that, for now, can only be calculated using dispersion approach:

$$a_\mu^{had}[LO] = \frac{1}{3}\left(\frac{\alpha}{\pi}\right)^2 \int_{m_\pi^2}^{\infty} \frac{K(s)}{s} R^{(0)}(s) = 6931(33)(7) \times 10^{-11}$$

The red part is due to experimental data taken from $\sigma(e^+e^- \to \text{hadrons})$.

Standard model result

QED (γ, ℓ)

$$a_\mu^{QED} = (116584718.951 \pm 0.009 \pm 0.019 \pm 0.007 \pm 0.077_\alpha) \times 10^{-11}$$

EW (W, Z)

$$a_\mu^{EW} = (154 \pm 1) \times 10^{-11}$$

Hadronic (quarks, gluons)

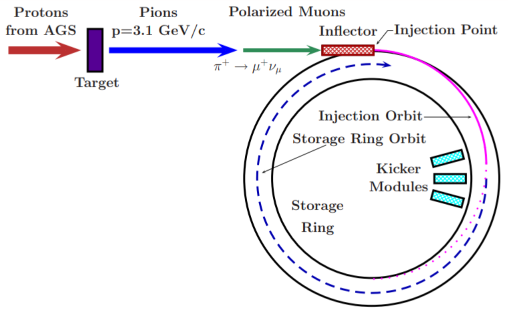
$$\begin{aligned} a_\mu^{HVP}[LO] &= (6923 \pm 42) \times 10^{-11} \\ a_\mu^{HVP}[HO] &= (-98.4 \pm 0.7) \times 10^{-11} \\ a_\mu^{HLbL} &= (105 \pm 26) \times 10^{-11} \end{aligned}$$

Total SM

$$a_\mu^{SM} = (116591828 \pm 50) \times 10^{-11}$$

What about the experiments?

General Principle of the experiments



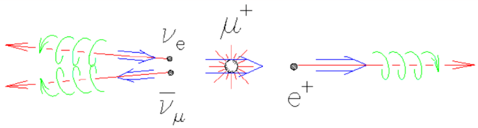
- Polarized muons are sent to the magnetic storage ring where they orbit and decay to positrons and neutrinos.
- As spin precesses around the magnetic field as a result decay positrons show modulations in their number.
- Decay positron oscillation is measured.

How do we polarize muons?

- It's a 2 body decay, neutrinos are left-handed.
- To conserve angular momentum, μ^+ has to be left-handed too, that is muon-spin directed opposite to its momentum.

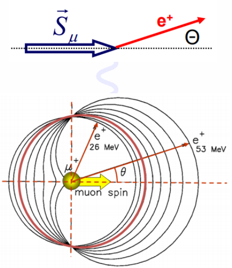
Can we measure their spin directions from the **decay** **positrons**?

Muon decay: Positron direction $\propto$ muon spin direction.



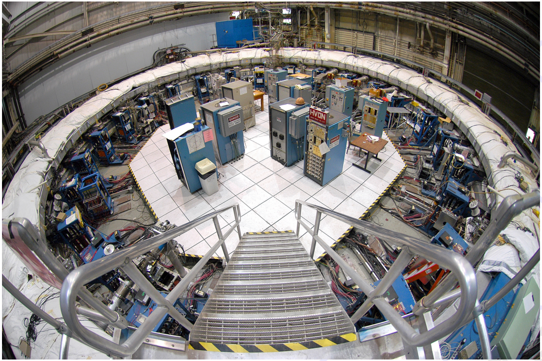
- 3-body decay, ν_e, $\bar{\nu}_\mu$ are left, right-handed respectively implying e^+ spin along its momentum.
- Due to parity violation, fast positrons are emitted along the muon spin direction.

$$W(\Theta) \propto [1 + \alpha(E) \cos\Theta]$$



μ^+ decay: highest energy e^+ along μ^+ spin

- Fastest e^+ are along the direction of muon spin.
- Therefore, detecting e^+ with energy $>$ a threshold, means reading the muon spin direction.
- That's why calorimeters are arranged near the beam path and pointed towards the beam.



How do we measure g-2?

Spin $\boldsymbol{s}$ when put in a magnetic field $\boldsymbol{B}$, precesses around it with a frequency

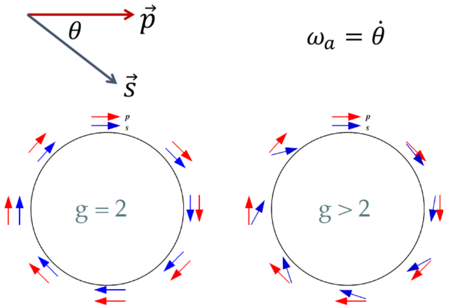
$$\omega_s = \frac{eB}{m_\mu}\left(\frac{1}{\gamma} + a_\mu\right)$$

But the muons are also orbiting inside the ring, this cyclotrone frequency is

$$\omega_c = \frac{eB}{m_\mu}\frac{1}{\gamma}$$

Therefore we define the difference as ω_a,

$$\omega_a = \frac{eB}{m_\mu} a_\mu$$



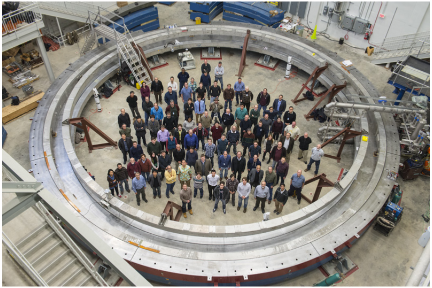
Remember $a_\mu = (g_\mu - 2)/2$

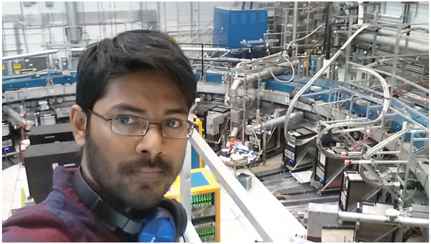
A **magic** worth mentioning...

To maintain vertical stability an electric field E (quadrupole) is also applied, which makes it a bit more complicated

$$\boldsymbol{\omega_a} = \frac{e}{m_\mu}\left(a_\mu \boldsymbol{B} - \left[a_\mu - \frac{1}{\gamma^2 - 1}\right]\boldsymbol{v} \times \boldsymbol{E}\right)$$

But there's a magic...





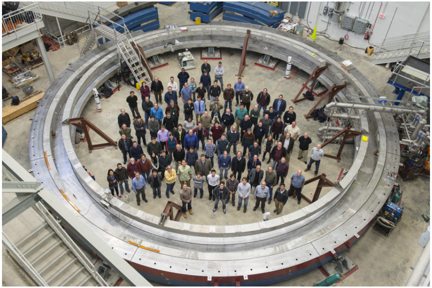
The **magic** momentum $\gamma \sim 29$

To maintain vertical stability an electric field E (quadrupole) is also applied, which makes it a bit more complicated

$$\boldsymbol{\omega_a} = \frac{e}{m_\mu}\left(a_\mu \boldsymbol{B} - \left[a_\mu - \frac{1}{\gamma^2 - 1}\right]\boldsymbol{v} \times \boldsymbol{E}\right)$$

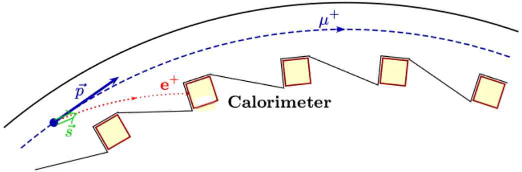
But there's a magic...
A clever choice of muon energy (3.1 GeV) or γ (~ 29) will result in a cancellation.

This specific momentum of the muon is called "**the magic-momentum**".



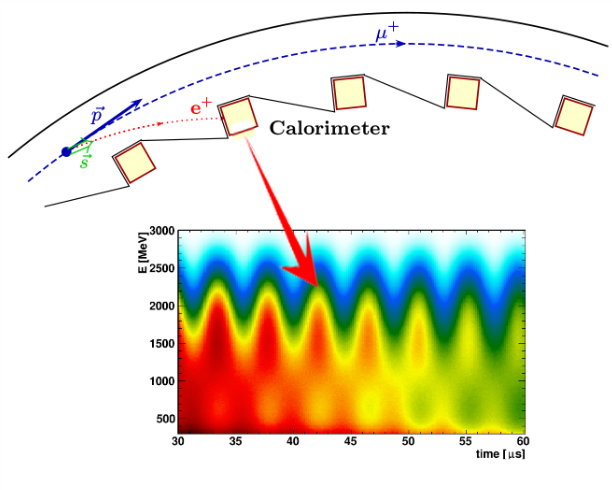
ω_a and decay positrons...

- $g_\mu = 2$ will imply $a_\mu = 0$ that is Larmor and cyclotrone frequencies will match perfectly implying no mis-alignment between the spin and the momentum of the muons.
- Slight mismatch between the two frequencies will show up in the misalignment between muon spin and momentum.
- This misalignment-oscillation will result in oscillation of the fast decay positrons.
- Measuring those decay positrons will mean measurement of ω_a hence a_μ.

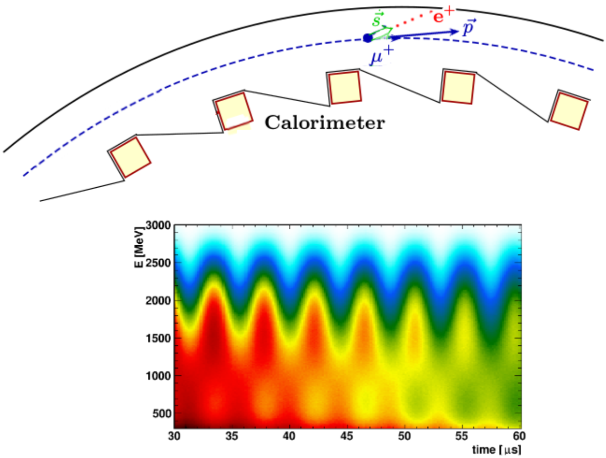


Remember muon spin and the decay positron energy are highly correlated!

The wiggles


μ^+
$\vec{p}$
e^+
$\vec{s}$
Calorimeter
3000
2500
2000
1500
1000
500
E [MeV]
30
35
40
45
50
55
60
time [μs]

The wiggles


$\vec{s}$
e^+
$\vec{p}$
μ^+
Calorimeter
E [MeV]
3000
2500
2000
1500
1000
500
30
35
40
45
50
55
60
time [μs]

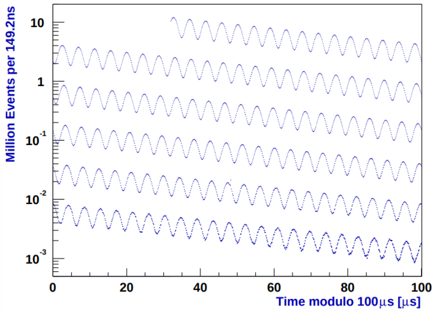
The wiggles and the ω_a

$$N(E,t) \propto e^{-t/\gamma\tau_\mu}\,[1 - A(E,t)\cos(\omega_a t + \phi)]$$

- Number of decay positrons N modulates with ω_a.
- It also exponentially decays with lifetime $\gamma\tau_\mu \sim 64$ μs.
- Recalling the equation

$$\omega_a = \frac{eB}{m_\mu} a_\mu$$

a measurement of the magnetic field and the muon mass will finish the job.

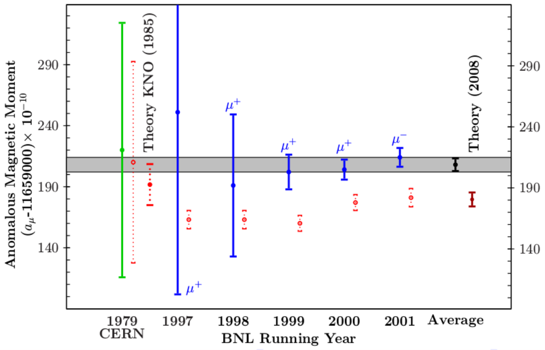


Exciting results!!!

All we discussed so far are common to most of the old experiments, let's consider the results and the consequences.

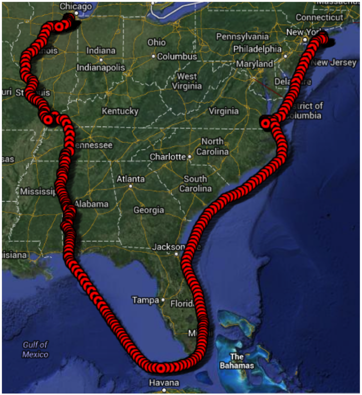
The Brookhaven (E821) result (0.54 ppm) is the last and the most exciting one, because

$$(a_\mu^{SM} - a_\mu^{expt}) \simeq \mathbf{3.6\sigma}$$



There's a 4-5 times more precise experiment going on right now... => an aim of $\mathbf{7}\sigma$ SM-experiment discrepancy.

The great move: 5150 kilometers, 25^{th} June $\rightarrow$ July 20^{th}, 2013



Arrival..

Celebration..

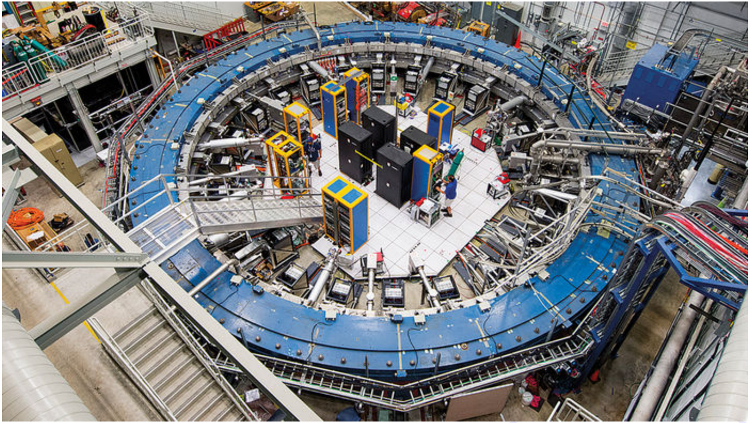
The **Fermilab** g-**2** collaboration

g-2 @ FNAL: Improvements of ***systematics*** over BNL

The new muon g-2 experiment at Fermilab, the E989 is aiming to be 4 times more precise ($0.14\ ppm$) than the BNL ($0.54\ ppm$), which requires improvements at several fronts.

- **Improved statistics:** $21\times$ BNL statistics.
- **Low pion contamination:** Pions (3.11 GeV) travels longer (~ 1 km) distance => pure muon beam at the end.
- **Gain calibration:** A **laser based calibration system** will monitor the changes in the gain during and outside a muon fill and correct for it.
- **Low pile-up:** A calorimeter is highly segmented (SiPMs collecting light from $\mathbf{6\times 9}$ PbF2 crystals) hence two hits separated even by $2\ ns$ can still be resolved.
- Improved tracker system.
- Extremely uniform and stable magnetic field (knowledge of 1.45 T B field at $\pm 7\ ppb$ level).

24 Calorimeters

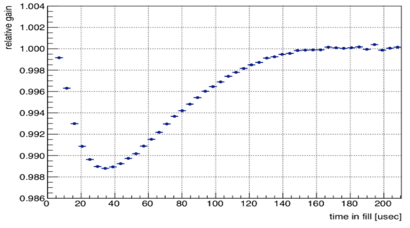


A little bit about the Italian contribution...

What we are doing: **Gain calibration**

Short-term gain change

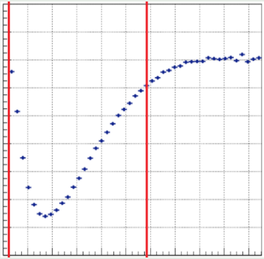
- Huge load during muon-fill in the ring causes the gains of the SiPMs drop significantly $\sim 10\%$
- Recovery time is typically a few **tens** of μs.



What we are doing: **Gain calibration**

Short-term gain change

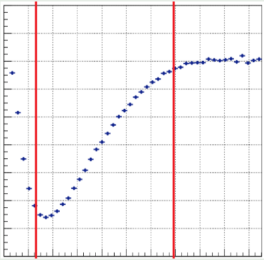
- We send 3 **laser pulses** of known intensity during a muon fill.



What we are doing: **Gain calibration**

Short-term gain change

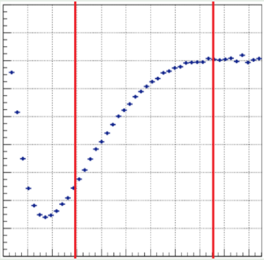
- We send 3 **laser pulses** of known intensity during a muon fill.
- In one of the next fills we shift those laser pulses by $5\ \mu s$.



What we are doing: **Gain calibration**

Short-term gain change

- We send 3 **laser pulses** of known intensity during a muon fill.
- In one of the next fills we shift those laser pulses by 5 μs.
- We continue shifting until we scan the whole "***gain-sagging***" muon fill window.



This way we obtain the "**gain-sagging function**" $G^{SiPM}(t)$ of the Silicon photo-multiplier.

$$r_{e^+} = r_{e^+}^{SiPM} \times G^{SiPM}$$

and correct the SiPM response.

What we are doing: **Gain calibration**

Long-term gain change

- Gain also varies with temperature, therefore day-night dependence is observed over longer DAQ time.
- We constantly send **laser pulses** of known intensity outside muon fills all the time to map the long term gain-change.

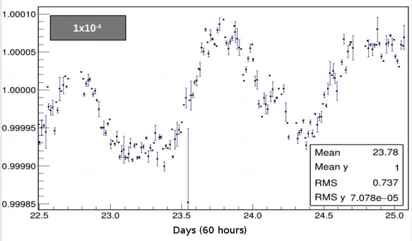


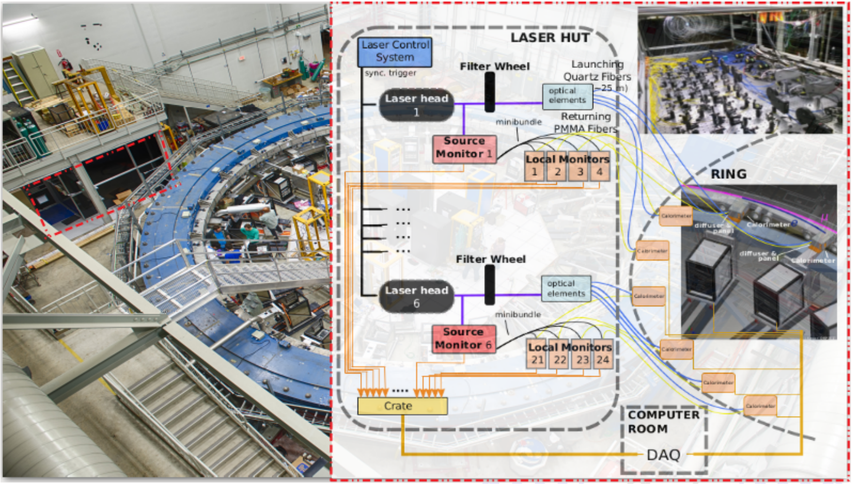
Figure: Ratio of known laser signals obtained using two Pin diodes over 60 hours.

This way we obtain the "**gain-sagging function**" $G^{SiPM}(t)$ of the Silicon photo-multiplier.

$$r_{e^+} = r_{e^+}^{SiPM} \times G^{SiPM}$$

and correct the SiPM response.

The laser calibration system



We are almost at the end... let's discuss the status of the experiment.

Current status and plans

- We have already achieved the BNL statistics.
- Results with 2× BNL data ($\sim 0.4\ ppm$) is expected to be published in the beginning of 2019.
- Final 20× BNL data ($\sim 0.14\ ppm$) is planned for 2021 publication.

If the central value of **BNL** stands, a **SM-experiment** discrepancy of $\mathbf{7}\sigma$ is expected!!!

Thanks

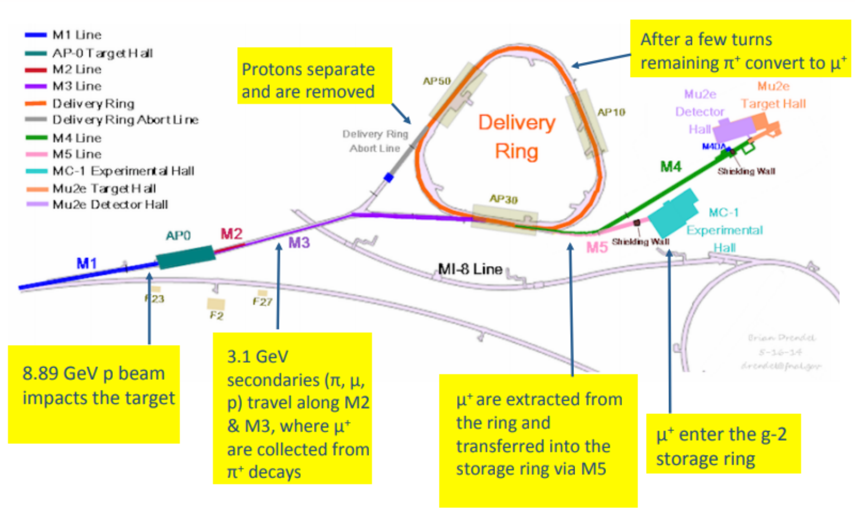
One of the biggest discrepancies in particle physics at the moment is the g-2 experiment. It's a measurement of the way the muon behaves in a magnetic field. The experiment shows a significant discrepancy with the Standard Model that's getting more significant with time. It's a low-profile experiment, but it's extremely sensitive to new physics. It's still running, but if I were to put my money on something that would signal new physics, it's the g-2 experiment at Fermilab. I think it's really fascinating.

-Brian Cox

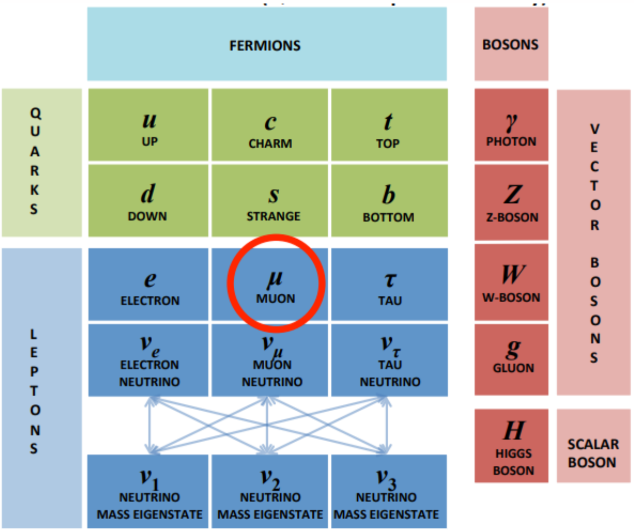
Some of the references

- *Muon Anomalous Magnetic Moment*, PDG
- *The anomalous magnetic moment of the muon: a theoretical introduction*, Marc Knecht
- *The Anomalous Magnetic Moment of the Muon*, Friedrich Jegerlehner
- *Experimental Prospects on Muon g-2*, Mark Lancaster

Backups


M1 Line
AP-0 Target Hall
M2 Line
M3 Line
Delivery Ring
Delivery Ring Abort Line
M4 Line
M5 Line
MC-1 Experimental Hall
Mu2e Target Hall
Mu2e Detector Hall
Protons separate and are removed
After a few turns remaining π⁺ convert to μ⁺
Delivery Ring Abort Line
AP50
AP10
AP30
Delivery Ring
Mu2e Detector Hall
Mu2e Target Hall
M4
Shielding Wall
MC-1 Experimental Hall
M5
Shielding Wall
APO
M2
M3
M1
MI-8 Line
F23
F2
F27
8.89 GeV p beam impacts the target
3.1 GeV secondaries (π, μ, p) travel along M2 & M3, where μ⁺ are collected from π⁺ decays
μ⁺ are extracted from the ring and transferred into the storage ring via M5
μ⁺ enter the g-2 storage ring

Backups


FERMIONS
BOSONS
QUARKS
LEPTONS
u
UP
c
CHARM
t
TOP
d
DOWN
s
STRANGE
b
BOTTOM
e
ELECTRON
μ
MUON
τ
TAU
ν_e
ELECTRON NEUTRINO
ν_μ
MUON NEUTRINO
ν_τ
TAU NEUTRINO
ν_1
NEUTRINO MASS EIGENSTATE
ν_2
NEUTRINO MASS EIGENSTATE
ν_3
NEUTRINO MASS EIGENSTATE
γ
PHOTON
Z
Z-BOSON
W
W-BOSON
g
GLUON
H
HIGGS BOSON
VECTOR BOSONS
SCALAR BOSON