

Magnetic Moment of Muons and New Physics

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On behalf of the Fermilab muon g-2 collaboration.

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One slide on me

- I am from Lala, Hailakandi.
- Did my B.Sc (Physics) from the Dept. of Physics, *G. C. College*, Silchar (2007).
- M.Sc from the *S. N. Bose National Centre for Basic Sciences*, Kolkata (2010).
- Two years of graduate school course work at *Tata Institute of Fundamental Research*, Mumbai after M.Sc.
- Ph.D, *University of Naples "Federico II"*, Naples, Italy, on modelling non-perturbative QCD techniques of the rare Kaon decays.
- Post Doctoral Fellow (Assegni di Ricerca), INFN (National Institute of Nuclear Physics), Naples, Italy in collaboration with Fermilab, USA.



What is this talk about?

What we do in science is the following:

- We see the apples fall and ask why?
- We develope a model that answers that "why".
- Next we measure how fast the apple falls, and compare with the number predicted by our **model**.
- If things don't match perfectly we improve our model... and so on.

Today's **apple** will be the **magnetic dipole moment** of a fundamental particle called **Muon** and our **model** will be the whole **physics**¹ that humanity has developed so far, this is knows as the **Standard Model**.

¹(except gravity)



Spin and magnetic moment of particles

Consider an electron orbiting in a circle of radius r with a speed v. The magnetic moment μ_{orb} will be given by the resulting current times the area of the circle:

$$\boldsymbol{\mu}_{orb} = i \boldsymbol{A} = \frac{-e}{2\pi r/v} \pi r^2 \, \hat{\boldsymbol{n}}$$

$$= -\frac{evr}{2} \times \frac{m}{m} \, \hat{\boldsymbol{n}} = -\frac{e}{2m} \boldsymbol{L}_{orb}$$

where $L_{orb} = r \times mv$.

This expression also holds for a fundamental particle with **intrinsic angular momentum** S and charge Q provided we correct it with a factor g:





Such a magnet when placed in a magnetic field experiences a torque that gives rise to a potential energy

$$V = -\mu \cdot B$$

Magnetic moment theory g-2 experiment The new g-2 experiment (E989) at Fermilab



The g factor

Pauli's equation, which is also the non-relativistic limit of **Dirac equation** describes a spin-1/2 particle ℓ with charge e_{ℓ} and mass m_{ℓ} in an external electromagnetic field given by the potentials (ϕ, \mathcal{A}) :

$$i \frac{\partial \psi}{\partial t} = \left[\underbrace{\frac{(-i \, \boldsymbol{\nabla} - \boldsymbol{e}_{\ell} \boldsymbol{\mathcal{A}})^2}{2m_{\ell}}}_{\text{Kinetic Energy}} - \underbrace{2 \frac{\boldsymbol{e}_{\ell}}{2m_{\ell}} \boldsymbol{S}_{\ell} \cdot \boldsymbol{B}}_{\text{Potential Energy}} + \boldsymbol{e}_{\ell} \boldsymbol{\phi}\right] \boldsymbol{\psi}$$

Comparing the second term (magnetic potential energy) with $U = -\mu \cdot B$ of the last slide, we see that Dirac's equation predicts

$$g=2$$

But is it??



History of $g \neq 2$

Lamb Shift (1947): According to relativistic quantum mechanics, g = 2, if that is true the energy difference between $2S_{1/2}$ and $2P_{1/2}$ levels of the Hydrogen atom should be zero.

Celebrated paper		
	PHYSICAL REVIEW VOLUME 72, NUMBER 3 AUGUST 1, 1947	
	Fine Structure of the Hydrogen Atom by a Microwave Method* **	
	WILLIS E. LAMI, JR. AND ROBERT C. RETHERFORD Columbia Radiation Laboratory, Department of Physics, Columbia University, New York, New York (Reveived June 18, 1947)	
	THE spectrum of the simplest atom, hydro- to the Dirac wave equation for an electronic due to electrons. Instead, we have found a method depending on a novel property of the moving in a Coulomb field is due to the combined effects of relativistic variation of mass with $\frac{1}{2}S_1$ level. According to the Dirac theory, this state exactly coincides in energy with the $2P_1$ velocity and spin-orbit coupling. It has been con- sidered one of the stress transfer of the two P states. The S	
	source one of the great transport of the same state in the absence of external electric helds is theory that if gave the "tight" fine structure of metastable. The radiative transition to the the energy levels. However, the experimental ground state $12S_1$ is forbidden by the selection	

But in the *Nobel* winning experiment, **Lamb** and **Retherford** found a relative shift of 1058 MHz suggesting $g \neq 2$! This result fueled the development of modern QED, the most successful theory in science so far.



RQM is not complete!

- Relativistic QM is not complete.
- Soon Schwinger, Tomonaga, Feynman and others started developing Quantum Electrodynamics (QED), that soon earned the reputation of the "most accurate theory of nature" so far and it still stands so.
- Feynman developed his diagramatic method that simplified super-complicated calculations of QED.

Let's learn Feynman Diagrams through Quantum Electrodynamics.

We will use the following conventions

- Time will flow from left to right.
- Verticle axis will roughly represent the space.





Fundamental propagators in QED



There's only **one allowed way** of joining them, the **QED vertex**, describes the interaction of a charged **particle** with an **EM** field (photon):



Following are not allowed





Let us draw the QED vertex in different ways

 \mathcal{M} and see what they mean.









• An electron emits a photon and moves on.









• A positron absorbs a photon and moves on.







M

• An electron and a positron annihilate into a photon.











• A photon produces an electron-positron pair.





Let us draw a bit more complicated diagrams with more than one vertex and involving **virtual particles**.



• Electron-positron pair annihilating to produce a photon that eventually produces another electron-postron pair.



Let us draw a bit more complicated diagrams with more than one vertex and involving **virtual particles**.





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C Fermilab

• Electron emits a photon and moves on, the photon eventually gets absorbed by a postron.



Let's draw a diagram with a virtual electron in a loop



A **photon** producing a virtual **electron-positron** pair that annihilates to re-produce the **photon**. This is basically the **photon interacting with the vacuum** of space which apparently is not so vacuum after all!

But as we go on **increasing the number of vertices**, the diagram becomes less and **less important**.

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QED: Lamb shift

At this moment we will be able to understand the lamb shift, the experiment that fueled the development of \mbox{QED} .

S and P orbitals have different shapes and gets kicked differently by virtual photons in the vacuum



as a result the energy of electrons in these two states differ slightly (0.00003%).



QED: magnetic moment of a particle.

We will be calculating a simple diagram of a **charged particle** interacting with an external **magnetic field**,



as this has the **minimum** number of **vertices**, this is the **dominant diagram** producing the result g = 2... But of course less important diagrams are possible representing the same **physical phenomenon**, that is diagrams with intermediate **virtual particles**, for example:



Complete result is the sum of all possible infinite number of diagrams.



Anomalous magnetic moment of the electron



First order correction (1-loop) was calculated by Julian Schwinger in 1948.







What's running in the loops?



That blob includes everything that is allowed in nature, leptons, quarks, weak-bosons or something unknown to current physics, any virtual field can interact with the lepton ℓ (that is running in a loop) in question and contribute to its g_{ℓ} factor. This is precisely why g_{ℓ} acts as an excellent probe to what's lurking in the vacuum, SM and/or BSM fields.

The anomaly is defined through the quantity $a_{\ell} = (g_{\ell} - 2)/2$. Total anomaly can be written as:

$$a_{\ell} = \underbrace{a_{\ell}^{QED} + a_{\ell}^{hadronic} + a_{\ell}^{weak}}_{\text{Standard Model}} + \underbrace{a_{\ell}^{BSM}}_{\text{New Physics}}$$
(1)

We need to know the SM contribution both theoretically and experimentally with equal precision in order to say something conclusive about the Beyond Standard Model part.



What's running in the loops?



Suppose a virtual particle of mass M_{loop} is running in the loop, contributions to the magnetic moment from such a loop enter as functions of m_{ℓ}^2/M_{loop}^2

$$a_\ell \sim f\left[rac{m_\ell^2}{M_{loop}^2}
ight]$$

therefore we can guess the mass of an unknown particle.

(

We are interested in the magnetic moment of a muon...



Standard Model of Physics





Quarks





Leptons







Charged Leptons

Atanu Nath





Neutrinos







Force Carriers













Muon





But why μ ? What's wrong with e?

As we have seen, loop contributions enter as functions of m_ℓ^2/M_{loop}^2

$$a_\ell \sim f\left[rac{m_\ell^2}{M_{loop}^2}
ight]$$

electron being the lightest lepton, even in a one-muon loop $M_{loop} = m_{\mu}$ implies (m_e^2/m_{μ}^2) a contribution $\mathcal{O}\left[10^{-10}\right]$.

Muon is ,

$$\frac{m_{\mu}^2}{m_e^2} \simeq 43000$$

times more sensitive than electron in sensing a heavy unknown particle.



What's wrong with τ then?

 τ is the heaviest lepton therefore m_τ^2/M_{loop}^2 is the biggest for τ , certainly a_τ will be the most sensitive probe to any new physics.

Well it is and the theoretical calculation provides us with a very precise value of

 $a_{\tau} = 117721(5) \times 10^{-8}$

but it is so short-lived $(10^{-13} \text{ seconds})$ that no practical experiment can be designed with the current technology, current experimental bound is

 $-0.052 < a_{\tau} < 0.013$

Even the sign is not known experimentally!!!

This leaves us with the only choice μ .



• QCD has this beautiful (yet nasty!) property called the "*assymptotic freedom*", strength of attraction between two quarks increases as we pull them apart.

Magnetic moment theory

 Which practically means, at long-distance (~ low-energy), diagrams with more vertices are more important in QCD!



- That means we have to calculate increasing number rof diagrams with increasing complexity!!! And there are infinite of them!
- The standard method called the **perturbation series** just fails, therefore scientists rely on **approximate models** for calculations as a result **big errors** enter the results.



The hadronic issue!



From left to right we have **leading order QED**, weak and hadronic contributions, biggest uncertainty of course enters from the hadronic (quark and gluon loops) contributions, that, for now, can only be calculated using dispersion approach:

$$a_{\mu}^{had}[LO] = \frac{1}{3} \left(\frac{\alpha}{\pi}\right)^2 \int_{m_{\pi}^2}^{\infty} \frac{K(s)}{s} R^{(0)}(s) = 6931(33)(7) \times 10^{-11}$$

The red part is due to experimental data taken from $\sigma(e^+e^- \rightarrow \text{hadrons})$.



Standard model result

QED (γ , ℓ)

$$a_{\mu}^{QED} = (116584718.951 \pm 0.009 \pm 0.019 \pm 0.007 \pm 0.077_{\alpha}) \times 10^{-11}$$
 EW (W, Z)

$$a_{\mu}^{EW} = (154 \pm 1) \times 10^{-11}$$

Hadronic (quarks, gluons)

$$\begin{array}{ll} a_{\mu}^{HVP}[LO] &= (6923 \pm 42) \times 10^{-11} \\ a_{\mu}^{HVP}[HO] &= (-98.4 \pm 0.7) \times 10^{-11} \\ a_{\mu}^{HLbL} &= (105 \pm 26) \times 10^{-11} \end{array}$$

Total SM

$$a_{\mu}^{SM} = (116591828 \pm 50) \times 10^{-11}$$



What about the experiments? Before we get there, we have to learn a few things:

- Spin and handed-ness of particles.
- Parity and its violation in Weak interactions.

Parity: this is like a **mirror**, if I am right-handed my mirror image will be a left-handed person. Parity operation is like taking a mirror image.



But what is **handed-ness** of a particle?

If the **spin** of a particle is directed along its **direction of motion** then the particle is **right-handed** otherwise it is a **left-handed** particle.



Parity violation in Weak interactions

Chinese-American lady **Chien-Shiung Wu** designed and led the famous **Wu-experiment** and found that **parity gets violated** in weak decays!

What does that mean?



Impications are profound

- physics is not the same if you look at this world and its mirrored version.
- some phenomena that you see in this world might not happen in the mirror world.

Let's have a brief look at her experiment.
Wu Experiment (1956)

Cobalt-60 decays to Nickel-60 emitting an electron and electron-anti-neutrino through weac interaction.

$$^{60}_{27} \mathrm{Co}
ightarrow ^{60}_{28} \mathrm{Ni} + e^- + \bar{
u}_e + 2\gamma$$

- On the right we have the real-wrold set-up, where the magnetic moment points up and electron is emitted at an angle θ.
- On the right we have the **mirror image** of the set-up, where the **magnetic moment points downwards**, emitted **electron** is expected at an angle $\pi \theta$ with the magnetic moment.

But they found that the angle θ is preferred over $\pi - \theta$ indicating an asymmetry between the real and the mirror world.





Mirro



Right-handed neutrinos don't exist!

Handedness: If the spin is directed along the direction of motion then it is right-handed otherwise it is a left-handed particle.

Consider a left-handed neutrino and and its mirror image, as the spin flips in the mirror, we should see a right-handed neutrino in the mirror. But we don't see any right-handed neutrinos in nature, meaning, nature doesn't always respect the mirror symmetry. Neutrinos are produced through **weak** interactions that prefers left over right.

- right-handed neutrinos don't exist.
- left-handed anti-neutrinos don't exist.



We are now ready to proceed with the muon g-2 experiment...



General Principle of the experiments



Polarized muons

are sent to the magnetic storage ring where they orbit and decay to positrons and neutrinos.

- As spin precesses around the magnetic field as a result decay positrons show modulations in their number.
- Decay positron oscillation is measured.

How do we polarize muons?

- It's a 2 body decay, neutrinos are left-handed.
- To conserve angular momentum, μ^+ has to be left-handed too, that is muon-spin directed opposite to its momentum.

Can we measure their spin directions from the decay positrons?



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Muon decay: Positron direction \propto muon spin direction.

 $\nu_{\rm e}$

- 3-body decay, ν_e , $\bar{\nu}_\mu$ are left, right-handed respectively implying e^+ spin along its momentum.
- Due to parity violation, fast positrons are emitted along the muon spin direction.

 $W(\Theta) \propto [1 + \alpha(E) \cos \Theta]$





μ^+ decay: highest energy e^+ along μ^+ spin

- Fastest e^+ are along the direction of muon spin.
- Therefore, detecting e^+ with energy $>{\rm a}$ threshold, means reading the muon spin direction.
- That's why calorimeters are arranged near the beam path and pointed towards the beam.





How do we measure g-2?

Spin s when put in a magnetic field B, precesses around it with a frequency

$$\omega_s = \frac{eB}{m_\mu} \left(\frac{1}{\gamma} + a_\mu \right)$$

But the muons are also orbiting inside the ring, this cyclotrone frequency is

$$\omega_c = \frac{eB}{m_\mu} \frac{1}{\gamma}$$

Therefore we define the difference as

 ω_a ,

$$\omega_a = \frac{eB}{m_\mu} a_\mu$$





A magic worth mentioning...

To maintain vertical stability an electric field E (quadrupole) is also applied, which makes it a bit more complicated

$$oldsymbol{\omega}_{oldsymbol{a}} = rac{e}{m_{\mu}} \left(a_{\mu} oldsymbol{B} - \left[a_{\mu} - rac{1}{\gamma^2 - 1}
ight] oldsymbol{v} imes oldsymbol{E}
ight)$$

But there's a magic...





The magic momentum $\gamma \sim 29$

To maintain vertical stability an electric field E (quadrupole) is also applied, which makes it a bit more complicated

$$\boldsymbol{\omega}_{\boldsymbol{a}} = rac{e}{m_{\mu}} \left(a_{\mu} \boldsymbol{B} - \left[rac{a_{\mu}}{\gamma^2 - 1} \right] \boldsymbol{v} \times \boldsymbol{E}
ight)$$

But there's a magic...

A clever choice of muon energy (3.1 GeV) or $\gamma~(\sim 29)$ will result in a cancellation.

This specific momentum of the muon is called "the magic-momentum".





ω_a and decay positrons...

- $g_{\mu} = 2$ will imply $a_{\mu} = 0$ that is Larmor and cyclotrone frequencies will match perfectly implying no mis-alignment between the spin and the momentum of the muons.
- Slight mismatch will result in oscillation of the number of fast decay positrons.
- Measuring those decay **positrons** will mean measurement of ω_a hence a_{μ} .



Remember muon spin and the decay positron energy are highly correlated!



The wiggles





The wiggles





The wiggles and the ω_a

$$N(E,t) \propto e^{-t/\gamma \tau_{\mu}} \left[1 - A(E,t) \cos(\omega_a t + \phi)\right]$$

- Number of decay positrons N modulates with ω_a .
- It also exponentially decays with lifetime $\gamma \tau_{\mu} \sim 64 \ \mu {\rm s}.$
- Recalling the equation

$$\omega_a = \frac{eB}{m_\mu} \, a_\mu$$

a measurement of the magnetic field and the muon mass will finish the job.





Before we discuss findings so far, let us try to understand what does a discovery mean in the particle physics language.



5 σ discovery

Consider a set of 60 dices with ormeroonone dice with all faces marked 3, but you don't know about this oddity!

- You expect to see one 3 in six rolls. That means if we roll 60 dices at once, we expect to see total of ten 3s.
- If you keep rolling all your dices you start to see spread around your expected result of ten 3s.



- But you see eleven 3s, five σ higher than your expected result!
- This is when you know with 99.9999% confidence that there's a strange dice in our set.

This is discovery!



Exciting results!!!

All we discussed so far are common to most of the old experiments, let's consider the results and the consequences.

The **Brookhaven** (E821) result (0.54 *ppm*) is the last and the most exciting one, because

 $(a_{\mu}^{SM} - a_{\mu}^{expt}) \simeq 3.6\sigma$





There's a 4-5 times more precise experiment going on right now... => an aim of 7σ theory-experiment disagreement.



The great move: 5150 kilometers, 25^{th} June \rightarrow July 20^{th} , 2013









Magnetic Moment of Muons and New Physics









Celebration..





The Fermilab g-2 collaboration





g-2 @ FNAL: Improvements of systematics over BNL

The new muon g-2 experiment at Fermilab, the E989 is aiming to be 4 times more precise $(0.14 \ ppm)$ than the BNL $(0.54 \ ppm)$, which requires improvements at several fronts.

- Improved statistics: 21× BNL statistics.
- Low pion contamination: Pions (3.11 GeV) travels longer ($\sim 1 \text{ km}$) distance => pure muon beam at the end.
- Gain calibration: A laser based calibration system will monitor the changes in the gain during and outside a muon fill and correct for it.
- Low pile-up: A calorimeter is highly segmented (SiPMs collecting light from 6 × 9 PbF2 crystals) hence two hits separated even by 2 *ns* can still be resolved.
- Improved tracker system.
- Extremely uniform and stable magnetic field (knowledge of 1.45 T B field at $\pm 7 \ ppb$ level).

Note: one ppm is an error of ~ 0.0001% or a certainty of 99.9999%, we are aiming for 10 times more precise measurement than that!





24 Calorimeters





A little bit about the Italian contribution...



What we are doing: Gain calibration

Short-term gain change

- Huge load during muon-fill in the ring causes the gains of the SiPMs drop significantly $\sim 10\%$
- Recovery time is typically a few **tens** of μ s.





What we are doing: Gain calibration

Short-term gain change

• We send 3 laser pulses of known intensity during a muon fill.





What we are doing: Gain calibration

Short-term gain change

- We send 3 laser pulses of known intensity during a muon fill.
- In one of the next fills we shift those laser pulses by 5 μ s.





What we are doing: Gain calibration

Short-term gain change

- We send 3 laser pulses of known intensity during a muon fill.
- In one of the next fills we shift those laser pulses by $5~\mu{\rm s}.$
- We continue shifting until we scan the whole "*gain-sagging*" muon fill window.



This way we obtain the "gain-sagging function" $G^{SiPM}(t)$ of the Silicon photo-multiplier.

$$r_{e^+} = r_{e^+}^{SiPM} \times G^{SiPM}$$

and correct the SiPM response.



What we are doing: Gain calibration

Long-term gain change

- Gain also varies with temperature, therefore day-night dependence is observed over longer DAQ time.
- We constantly send laser pulses of known intensity outside muon fills all the time to map the long term gain-change.



Figure: Ratio of known laser signals obtained using two Pin diodes over 60 hours.

This way we obtain the "gain-sagging function" $G^{SiPM}(t)$ of the Silicon photo-multiplier.

$$r_{e^+} = r_{e^+}^{SiPM} \times G^{SiPM}$$

and correct the SiPM response.



The laser calibration system





We are almost at the end... let's discuss the status of the experiment.





Current status and plans

- We have already achieved the Brookhaven statistics.
- Results with $2 \times$ BNL data (certainty ~ 99.99996%) is expected to be published in the beginning of 2019.
- Final $20 \times BNL$ data ((certainty ~ 99.999996%)) is planned for 2021 publication.

If the central value of BNL stands, a SM-experiment discrepancy of 7σ is expected, which is 2 σ more than what we need for a discovery!!!





Thanks



One of the biggest discrepancies in particle physics at the moment is the <u>g-2</u> <u>experiment</u>. It's a measurement of the way the muon behaves in a magnetic field. The experiment shows a significant discrepancy with the Standard Model that's getting more significant with time. It's a low-profile experiment, but it's extremely sensitive to new physics. It's still running, but if I were to put my money on something that would signal new physics, it's the g-2 experiment at Fernilab. I think it's really fascinating.

-Brian Cox



Some of the references

- Muon Anomalous Magnetic Moment, PDG
- The anomalous magnetic moment of the muon: a theoretical introduction, Marc Knecht
- The Anomalous Magnetic Moment of the Muon, Friedrich Jegerlehner
- Experimental Prospects on Muon g-2, Mark Lancaster





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Magnetic moment theory g-2 experiment The new g-2 experiment (E989) at Fermilab

Backups




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Backups

